UNIT 1 ASTRONOMICAL SCALES

Structure

1.1 INTRODUCTION

You have studied in Units 9 to 11 of the Foundation Course in Science and Technology (FST-1) that the universe is vast. You know that the Sun is one amongst billions of stars situated in as many galaxies. You have also learnt that the distances between planets and stars are huge, and so are their masses. For example, the distance between the Sun and the Earth is of the order of 1.5×10^{11}m. The radius of the Sun itself is about 7×10^{8}m, which is almost 100 times the Earth's radius. The mass of the Earth is of the order of 10^{24}kg and the Sun's mass is a million times larger. The time scales involved are also huge. For example, the estimated age of the Sun is about 5 billion years, compared to the lifetime of a human being, which is less than 100 years in most cases. All these numbers are very large compared to the lengths, masses and time scales we encounter everyday. Obviously, we need special methods to measure them and represent them.

The distances and masses of celestial objects are of fundamental interest to astronomy. Does a star in the night sky seem bright to us because it is closer, or is it so because it is intrinsically bright? The answer can be obtained if we know the distance to a star. You have also learnt in Unit 10 of FST-1 that the mass of a star determines how it will evolve.

In this unit we introduce you to some important physical quantities of interest in astronomy, such as *distance*, *size*, *mass*, *time*, *brightness*, *radiant flux*, *luminosity*, *temperature* and their scales. You will also learn about some simple methods of measuring these quantities. In the next unit, you will learn about the various coordinate systems used to locate the positions of celestial objects.

Objectives

After studying this unit, you should be able to:

- describe the distance, mass, time and temperature scales used in astronomy and astrophysics;
- compare the brightness and luminosity of astronomical objects; and
- determine the distance, size and mass of astronomical objects from given data.

1.2 ASTRONOMICAL DISTANCE, MASS AND TIME SCALES

In astronomy, we are interested in measuring various physical quantities, such as mass, distance, radius, brightness and luminosity of celestial objects. You have just learnt that the scales at which these quantities occur in astronomy are very different from the ones we encounter in our day-to-day lives.

Therefore, we first need to understand these scales and define the units of measurement for important astrophysical quantities.

We begin with astronomical distances.

Astronomical Distances

You have studied in your school textbooks that the Sun is at a distance of about 1.5×10^{11}m from the Earth. The mean distance between the Sun and the Earth is called **one astronomical unit**. Distances in the solar system are measured in this unit.

Another unit is the **light year**, used for measuring distances to stars and galaxies.

The **parsec** is a third unit of length measurement in astronomy.

We now define them.

Units of measurement of distances

- 1 **Astronomical Unit** (AU) is the mean distance between the Sun and the Earth.

  $$\mathbf{1\ AU = 1.496 \times 10^{11} m}$$

- 1 **Light Year** (ly) is the distance travelled by light in one year.

  $$\mathbf{1\ ly = 9.460 \times 10^{15} m = 6.323 \times 10^{4}\ AU}$$

- 1 **Parsec** (pc) is defined as the distance at which the radius of Earth's orbit subtends an angle of 1″ (see Fig.1.1).

  $$\mathbf{1\ pc = 3.262\ ly = 2.062 \times 10^{5} AU = 3.085 \times 10^{16} m}$$

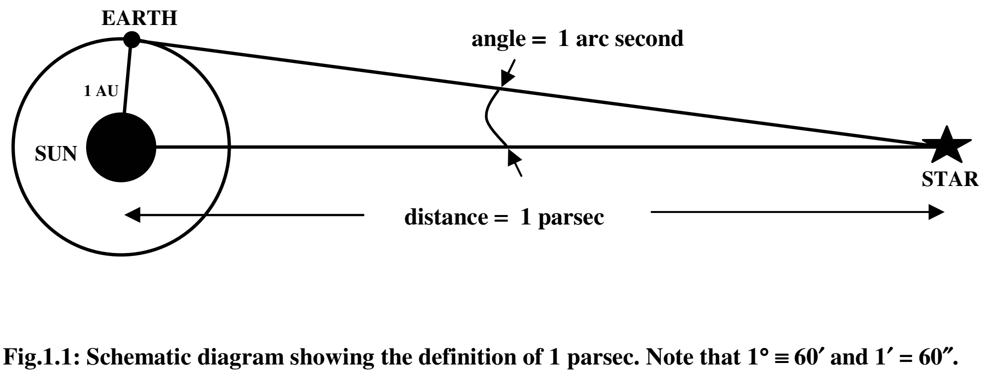


Fig.1.1: Schematic diagram showing the definition of 1 parsec. Note that 1° ≡ 60′ and 1′ = 60″. Thus, 1″ = 1/3600 degree

Dimensions of Astronomical Objects

The sizes of stars or stellar dimensions are usually measured in units of solar radius R_Θ. For example, Sirius (लुब्धक), the brightest star in the sky, has radius $2R_\Theta$. The radius of the star Aldebaran (रोहिणी) in Taurus is $40R_\Theta$ and that of Antares (ज्येष्ठा) in Scorpius is 700 R_Θ.

> **Unit of measurement of size**
>
> **1 solar radius, $R_\Theta = 7 \times 10^8$m**

Mass

Stellar masses are usually measured in units of solar mass M_Θ. We know that $M_\Theta = 2 \times 10^{30}$ kg. For example, the mass of our galaxy is ~ 10^{11} M_Θ. The mass of a globular cluster is of the order of $10^5 - 10^6$ M_Θ. S. Chandrasekhar showed (Unit 11) that the mass of a white dwarf star cannot exceed 1.4 M_Θ. This is called the **Chandrasekhar limit**.

> **Unit of measurement of mass**
>
> **1 solar mass $M_\Theta = 2 \times 10^{30}$ kg**

Time Scales

The present age of the Sun is about 5 billion years. It has been estimated that it would live for another 5 billion years in its present form. The age of our galaxy may be around 10 billion years. Various estimates of the age of the universe itself give a figure between 12 and 16 billion years. On the other hand, if the pressure inside a star is insufficient to support it against gravity, then it may collapse in a time, which may be measured in seconds, rather than in millions of years.

In Table 1.1, we list the distances, sizes and masses of some astronomical objects.

Table 1.1: Distance, radii and masses of astronomical objects

	Distance	Radius	Mass	Remarks
Sun	1 AU	1 R_Θ	1 M_Θ	–
Earth	–	0.01 R_Θ	10^{-6} M_Θ	–
Jupiter	4 AU (5 AU from the Sun)	0.1 R_Θ	10^{-3} M_Θ	Largest planet
Proxima Centauri	1.3 pc	0.15 R_Θ	0.12 M_Θ	Nearest star
Sirius A	2.6 pc	2 R_Θ	3 M_Θ	Brightest star
Sirius B	2.6 pc	0.02 R_Θ	1 M_Θ	First star identified as white dwarf
Antares	150 pc	700 R_Θ	15 M_Θ	Super giant star

You may like to express the distances and sizes of some astronomical objects in various units introduced here.

Spend 10 *min*.

SAQ 1

a) Express the distance between Jupiter and Sun in parsecs, and the distance between the Earth and the Sun in light years.

b) Express the radius of the Earth in units of $R_{\odot}$.

Next time when you look at the familiar stars in the night sky, you will have some idea of how far these are from us, and also how massive they are.

An important problem in astronomy is to find out how much energy is emitted by celestial objects. It is expressed in terms of the luminosity and is related to the radiant flux and brightness of the object. You may have noticed that some stars in the night sky appear bright to us, some less bright and others appear quite faint. How do we estimate their real brightness? Let us find out.

1.3 BRIGHTNESS, RADIANT FLUX AND LUMINOSITY

It is a common experience that if we view a street lamp from nearby, it may seem quite bright. But if we see it from afar, it appears faint. Similarly, a star might look bright because it is closer to us. And a really brighter star might appear faint because it is too far. We can estimate the apparent brightness of astronomical objects easily, but, if we want to measure their **real or intrinsic brightness**, we must take their distance into account. The **apparent brightness** of a star is defined in terms of what is called the **apparent magnitude** of a star.

Apparent Magnitude

In the second century B.C., the Greek astronomer Hipparchus was the first astronomer to catalogue stars visible to the naked eye. He divided stars into six classes, or **apparent magnitudes**, by their relative brightness as seen from Earth. He numbered the apparent magnitude (m) of a star on a scale of 1 (the brightest) to 6 (the least bright). This is the scale on which the apparent brightness of stars, planets and other objects is expressed as they appear from the Earth. The **brightest** stars are assigned the **first magnitude** ($m = 1$) and the **faintest stars** visible to the naked eye are assigned the **sixth magnitude** ($m = 6$).

> **Apparent Magnitude**
>
> ***Apparent magnitude*** **of an astronomical object is a measure of how bright it** ***appears.*** **According to the magnitude scale**, ***a smaller magnitude means a brighter star*****.**

The magnitude scale is actually a *non-linear* scale. What this means is that a star, two magnitudes fainter than another, is not twice as faint. Actually it is about 6.3 times fainter. Let us explain this further.

The response of the eye to increasing brightness is nearly logarithmic. We, therefore, need to define a **logarithmic scale for magnitudes** in which **a difference of 5 magnitudes is equal to a factor of 100 in brightness**. On this scale, the brightness ratio corresponding to 1 magnitude difference is $100^{1/5}$ or 2.512.

Therefore, a star of magnitude 1 is 2.512 times brighter than a star of magnitude 2.

It is $(2.512)^2 = 6.3$ times brighter than a star of magnitude 3.

How bright is it compared to stars of magnitude 4 and 5?

It is $(2.512)^3 = 16$ times brighter than a star of magnitude 4.

And $(2.512)^4 = 40$ times brighter than a star of magnitude 5.

As expected, it is $2.512^5 = 100$ times brighter than a star of magnitude 6.

For example, the pole star (Polaris, *Dhruva*) has an apparent magnitude +2.3 and the star Altair has apparent magnitude 0.8. Altair is about 4 times brighter than Polaris.

Mathematically, the brightness b_1 and b_2 of two stars with corresponding magnitudes m_1 and m_2 are given by the following relations.

Relationship between brightness and apparent magnitude

$$m_1 - m_2 = 2.5\log_{10}\left(\frac{b_2}{b_1}\right) \tag{1.1}$$

$$\frac{b_2}{b_1} = 100^{(m_1 - m_2)/5} \qquad \frac{b_1}{b_2} = 100^{-(m_1 - m_2)/5} \tag{1.2}$$

In Table 1.2, we give the brightness ratio for some magnitude differences.

Table 1.2: Brightness ratio corresponding to given magnitude difference

Magnitude Difference	Brightness Ratio
0.0	1.0
0.2	1.2
1.0	2.5
1.5	4.0
2.0	6.3
2.5	10.0
3.0	16.0
4.0	40.0
5.0	100.0
7.5	1000.0
10.0	10000.0

Modern astronomers use a similar scale for apparent magnitude. With the help of telescopes, a larger number of stars could be seen in the sky. Many stars fainter than the 6^{th} magnitude were also observed. Moreover, stars brighter than the first magnitude have also been observed.

Thus a magnitude of zero or even negative magnitudes have been assigned to extend the scale. A star of −1 magnitude is 2.512 times brighter than the star of zero magnitude. The brightest star in the sky other than the Sun, Sirius A, has an apparent magnitude of − 1.47.

The larger magnitude on negative scale indicates higher brightness while the larger positive magnitudes indicate the faintness of an object.

The faintest object detectable with a large modern telescope in the sky currently is of magnitude $m = 29$.

Therefore, the Sun having the apparent magnitude $m = -26.81$, is 10^{22} times brighter than the faintest object detectable in the sky.

In the following table we list the apparent magnitudes of some objects in the night sky.

Table 1.3: Apparent magnitudes of some celestial objects

Object	Indian Name	Apparent Magnitude
Sun	*Surya*	−26.81
Full Moon	*Chandra*	−12.73
Venus	*Shukra*	−4.22
Jupiter	*Guru*	−2.60
Sirius A	*Vyadha*	−1.47
Canopus	*Agastya*	−0.73
α-Centauri		−0.10
Betelgeuse	*Ardra*	+0.80
Spica	*Chitra*	+0.96
Polaris	*Dhruva*	+2.3
Uranus	*Varuna*	+5.5
Sirius B		+8.68
Pluto		+14.9
Faintest Star (detected by a modern telescope)		+29

Let us now apply these ideas to a concrete example.

Example 1: Comparison of Brightness

Compare the brightness of the Sun and α-Centauri using the apparent magnitudes listed in the Table 1.3.

Solution

From Table 1.3, $m_{Sun} - m_{\alpha C} = -26.81 - (-0.10) = -26.71$. Therefore, using Eq. (1.2), we obtain

$$\frac{b_{\alpha C}}{b_{Sun}} = 100^{-(26.71)/5} = 10^{-10.7}$$

or

$$\frac{b_{Sun}}{b_{\alpha C}} = 10^{10.7}$$, i.e. the Sun is about 10^{11} times brighter than α-Centauri.

You may now like to solve a problem based on what you have studied so far.

SAQ 2

Spend 10 min.

a) The apparent magnitude of the Sun is – 26.81 and that of the star Sirius is – 1.47. Which one of them is brighter and by how much?

b) The apparent magnitudes of the stars Arcturus and Aldebaran are 0.06 and 0.86, respectively. Calculate the ratio of their brightness.

The apparent magnitude and brightness of a star do not give us any idea of the total energy emitted per second by the star. This is obtained from **radiant flux** and the **luminosity** of a star.

Luminosity and Radiant Flux

The *luminosity* of a body is defined as the total energy radiated by it per unit time.

***Radiant flux* at a given point is the total amount of energy flowing through per unit time per unit area of a surface oriented normal to the direction of propagation of radiation.**

The unit of radiant flux is erg s^{-1} cm^{-2} and that of luminosity is erg s^{-1}.

In astronomy, it is common to use the cgs system of units. However, if you wish to convert to SI units, you can use appropriate conversion factors.

Note that here the radiated energy refers to not just visible light, but includes all wavelengths.

The radiant flux of a source depends on two factors:

(i) the radiant energy emitted by it, and

(ii) the distance of the source from the point of observation.

Suppose a star is at a distance r from us. Let us draw an imaginary sphere of radius r round the star. The surface area of this sphere is $4\pi r^2$. Then the radiant flux F of the star, is related to its luminosity L as follows:

$$F = \frac{L}{4\pi r^2} \tag{1.3}$$

The luminosity of a stellar object is a measure of the intrinsic brightness of a star. It is expressed generally in the units of the solar luminosity, L_Θ, where

$$L_\Theta = 4\times10^{26}\,\mathrm{W} = 4\times10^{33}\,\mathrm{erg\,s^{-1}}$$

For example, the luminosity of our galaxy is about 10^{11} L_Θ.

Now, the energy from a source received at any place, determines the brightness of the source. This implies that F is related to the brightness b of the source: the brighter the source, the larger would be the radiant flux at a place. Therefore, the ratio of brightness in Eq. (1.2) can be replaced by the ratio of radiant flux from two objects at the same place and we have

$$\frac{F_2}{F_1} = 100^{(m_1-m_2)/5} \tag{1.4}$$

You know from Eq. (1.3) that the flux received at a place also depends on its distance from the source. Therefore, two stars of the same apparent magnitude may not be equally luminous, as they may be located at different distances from the observer: A star's apparent brightness does not tell us anything about the luminosity of the star. We need a measure of the **true** or **intrinsic brightness** of a star. Now, we could easily compare the true brightness of stars if we could line them all up at the same distance from us (see Fig. 1.2). With this idea, we define the **absolute magnitude** of a star as follows:

Absolute Magnitude

The *absolute magnitude*, *M*, of an astronomical object is defined as its apparent magnitude if it were at a distance of 10 pc from us.

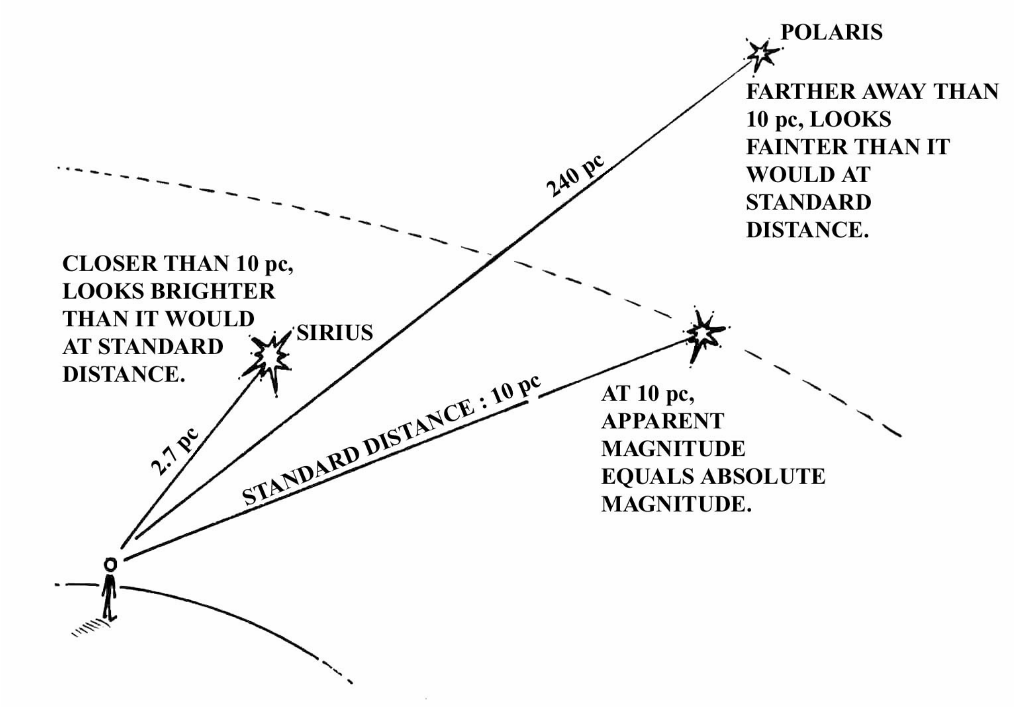


Fig.1.2: Absolute magnitude of astronomical objects

Let us now relate the absolute magnitude of a star to its apparent magnitude. Let us consider a star at a distance *r* pc with apparent magnitude *m*, intrinsic brightness or luminosity *L* and radiant flux F_1. Now when the same star is placed at a distance of 10 pc from the place of observation, then its magnitude would be *M* and the corresponding radiant flux would be F_2. From Eq. (1.4), we have

$$\frac{F_2}{F_1} = 100^{(m-M)/5} \tag{1.5}$$

Since the luminosity is constant for the star, we use Eq. (1.3) to write

$$\frac{F_2}{F_1} = \left(\frac{r\ \text{pc}}{10\ \text{pc}}\right)^2 \tag{1.6}$$

Using Eq. (1.6) in Eq. (1.5), we get the difference between the apparent magnitude (*m*) and absolute magnitude (*M*).

It is a measure of distance and is called the **distance modulus** (see Fig. 1.3).

Distance modulus

$$m - M = 5\log_{10}\left(\frac{r\,\text{pc}}{10\,\text{pc}}\right) = 5\log_{10} r - 5 \qquad (1.7)$$

Fig.1.3: Star cluster showing distance modulus as a measure of distance. For the star farther away, $m = 12.3$, $M = 2.6$, $r = 871$ pc. For the closer star, $m = 8.0$, $M = 5.8$, $r = 28$ pc

We can also relate the absolute magnitudes of stars to their luminosities. From Eq. (1.3), we know that the ratio of radiant flux of two stars **at the same distance from the point of observation** is equal to the ratio of their luminosities. Thus, if M_1 and M_2 are the absolute magnitudes of two stars, using Eq. (1.5), we can relate their luminosities to M_1 and M_2.

Relationship between Luminosity and Absolute Magnitude

$$\frac{L_2}{L_1} = 100^{(M_1 - M_2)/5} \qquad (1.8)$$

or

$$M_1 - M_2 = 2.5\log_{10}\left(\frac{L_2}{L_1}\right) \qquad (1.9)$$

Thus, the absolute magnitude of a star is a measure of its luminosity, or intrinsic brightness.

Often if we know what kind of star it is, we can estimate its absolute magnitude. We can measure its apparent magnitude (m) directly and solve for distance using Eq. (1.7). For example, the apparent magnitude of Polaris (pole star) is +2.3. Its absolute magnitude is –4.6 and it is 240 pc away. The apparent magnitude of Sirius A is –1.47, its absolute magnitude is +1.4 and it is at a distance of 2.7 pc.

You may now like to stop for a while and solve a problem to fix these ideas.

SAQ 3

Spend 5 min.

a) The distance modulus of the star Vega is –0.5. At what distance is it from us?

b) If a star at 40 pc is brought closer to 10 pc, i.e., 4 times closer, how bright will it appear in terms of the magnitude?

We now discuss some simple methods of measuring astronomical distances, sizes, masses and temperatures.

1.4 MEASUREMENT OF ASTRONOMICAL QUANTITIES

Since the brightness of heavenly objects depends on their distances from us, the measurement of distance is very important in astronomy. You must have measured the lengths of several objects in your school and college laboratories. But how do we measure astronomical distances? Obviously, traditional devices like the metre stick or measuring tapes are inadequate for such measurements. Other less direct ways need to be used. We now discuss some common methods of measuring astronomical distances. Since stars have been studied most extensively, we will focus largely on them in our discussion.

1.4.1 Astronomical Distances

You may be familiar with the method of trigonometric parallax. To get an idea of what it is, perform the following activity.

Activity 1: Trigonometric Parallax

Extend your arm and hold your thumb at about one foot or so in front of your eyes. Close your right eye and look at your thumb with your left eye. Note its position against a distant background. Now close your left eye and look at your thumb with your right eye. Do you notice that the position of the thumb has shifted with respect to the background? Your thumb has not moved. However, since you have looked at it from different point (left and right eyes), it **seems** to have shifted. The shift in the apparent position of the thumb can be represented by an angle θ (Fig. 1.4).

Parallax is the apparent change in the position of an object due to a change in the location of the observer.

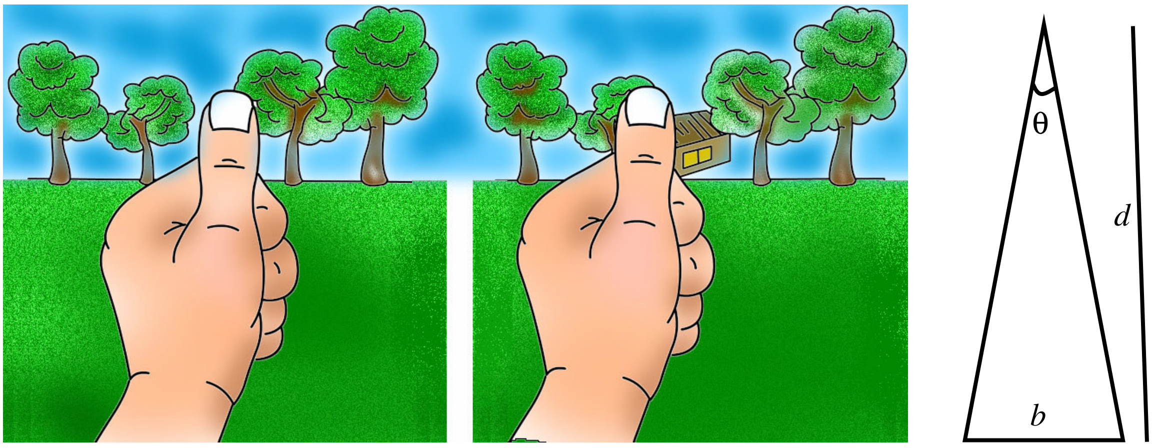


Fig.1.4: Parallax angle and baseline

We call θ/2, the **parallax angle**. The distance b between the points of observation (in this case your eyes), is called the **baseline**. From simple geometry, for small angles, $\frac{\theta}{2} = \frac{b}{d}$, where d is the distance from the eyes to the thumb.

The parallax method can be used to measure the distances of stars and other objects in the sky. The principle of the method is similar to the one used in finding the height of mountain peaks, tall buildings, etc.

Let us now find out how this method can be used to measure astronomical distances.

Stellar Parallax

For measuring the distance of a star, we must use a very long baseline. Even for measuring the distance to the nearest star, we require a baseline length greater than the Earth's diameter. This is because the distance of the star is so large that the angle measured from two diametrically opposite points on the Earth will differ by an amount which cannot be measured. Therefore, we take the **diameter of the Earth's orbit** as the baseline, and **make two observations** at an interval of six months (see Fig. 1.5).

One half of the maximum change in angular position (Fig. 1.5) of the star is defined as its **annual parallax**. From Fig. 1.5, the distance r of the star is given by

$$\frac{d_{SE}}{r} = \tan\theta \qquad (1.10a)$$

where d_{SE} is the average distance between the Sun and the Earth. Since the angle θ is very small, $\tan\theta \cong \theta$, and we can write

$$r = \frac{d_{SE}}{\theta} \qquad (1.10b)$$

Remember that this relation holds only when the parallax angle θ is expressed in radians.

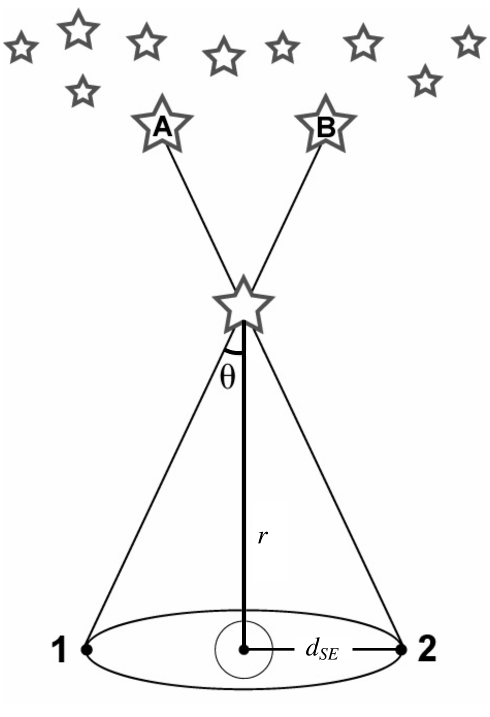


Fig.1.5: Stellar parallax

Since, $d_{SE} = 1\,AU$, we have

$$r = \frac{1AU}{\theta} \qquad (1.10c)$$

If we measure θ in arc seconds, then the distance is said to be in **parsecs**.

> **One parsec is the distance of an object that has a *par*allax of one *sec*ond of an arc (1″).**

The nearest star Proxima Centauri has a parallax angle 0.77 ″. Thus its distance is 1.3 pc. Since the distance is proportional to 1/θ, the more distant a star is, the smaller is its parallax.

In Table 1.4 we give the parallax angles and distances of some stars.

Table 1.4: Parallaxes and distances of some bright stars

Star	θ (in arc-seconds)	distance (*r* pc)
α-CMa	0.375	2.67
αCMi	0.287	3.48
αAquila	0.198	5.05
αTauri	0.048	20.8
αVirginis	0.014	71.4
αScorpii	0.008	125

Note that the angle θ cannot be measured precisely when the stellar object is at a large distance. Therefore, alternative methods are used to determine distances of stellar objects.

You could now try an exercise to make sure you have grasped the concept of parallax.

Spend 10 *min.*

SAQ 4

a) The parallax angles of the Sun's neighbouring stars (in arc-seconds) are given below. Calculate their distances.

Star	Parallax
Alpha Centauri	0.745
Barnard's star	0.552
Altair	0.197
Alpha Draco	0.176

b) A satellite measures the parallax angle of a star as 0.002 arc-second. What is the distance of the star?

You have just learnt that the parallax method helps us in finding the distances to nearby stars. But how can we find out which stars are nearby? We can do this by observing the motion of stars in the sky over a period of time.

Proper Motion

All celestial objects, the Sun, the Moon, stars, galaxies and other bodies are in relative motion with respect to one another. Part of their relative motion is also due to the Earth's own motion. However, the rate of change in the position of a star is very slow. It is not appreciable in one year or even in a decade. For example, if we photograph a small area of the sky at an interval of 10 years, we will find that some of the stars in the photograph have moved very slightly against the background objects (Fig.1.6).

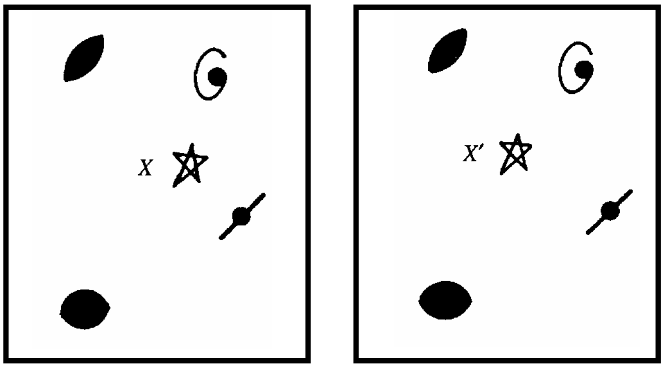


Fig.1.6: Motion of a star with respect to distant background objects

The motion of a star can be resolved along two directions:

i) Motion along the line of sight of the observer, (either towards or away from the observer) is called the **radial motion**.

ii) Motion perpendicular to the line of sight of the observer is called **proper motion** (see Fig. 1.7).

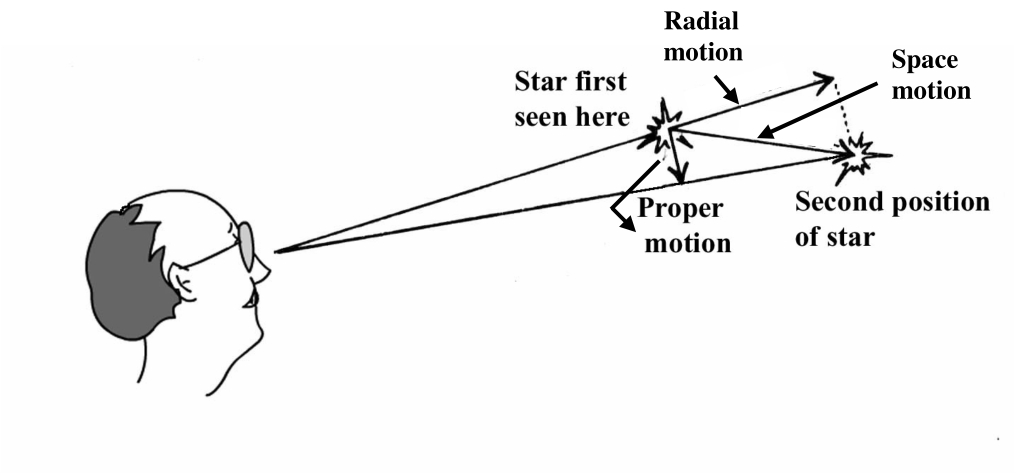


Fig.1.7: Radial and proper motion of a star

Radial motion causes the spectral lines of a star to shift towards red (if the motion is away from the observer) or towards blue (if the motion is towards the observer). This shift is the well-known *Doppler shift*. The proper motion is very slow. It is measured over an interval of 20 to 30 years. It is expressed in arc seconds per year. The average proper motion for all naked eye stars is less than 0.1 arc second/yr.

The proper motion is denoted by μ. For a star at a distance r from the Earth it is related to its transverse velocity as follows:

$$\text{proper motion} = \frac{\text{transverse velocity}}{\text{distance of the star}}$$

or $$\mu = \frac{v_\theta}{r}, \qquad (1.11a)$$

where v_θ is the transverse velocity.

Hence,

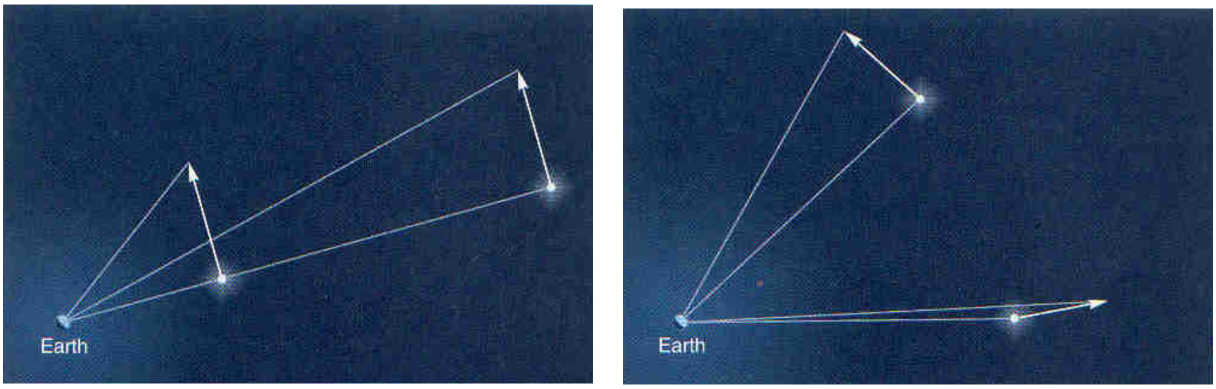
$$v_\theta = \mu r \qquad (1.11b)$$

If μ is measured in units of arc-seconds per year and r in pc, the transverse velocity is given by

$$v_\theta(\text{km s}^{-1}) = 4.74\mu r \qquad (1.11c)$$

If we add the radial velocity vector and the proper motion vector, we obtain the **space velocity** of a star (Fig.1.7).

We can locate stars that are probably nearby by looking for stars with large proper motions (see Fig. 1.8a). Proper motion of a star gives us statistical clues to its distance. If we see a star with a small proper motion, it is most likely to be a distant star. However, we cannot be absolutely certain since it could also be a nearby star moving directly away from us or toward us (see Fig. 1.8b).



(a) **(b)**

Fig.1.8: a) If two stars have the same space velocity and move perpendicular to the line of sight, the one with the larger proper motion will be nearer; b) Two stars at the same distance with the same velocity may have different proper motions, if one moves perpendicular to the line of sight and the other is nearly parallel to the line of sight

We know that the Sun itself is not stationary. The space velocity vector of a star must be corrected by subtracting from it the velocity vector of the Sun.

The space velocity of a star corrected for the motion of the Sun is termed as the **peculiar velocity** of the star.

The peculiar velocities of stars are essentially random and their typical magnitude is such that in a time of about 10^6 years the shape of the present constellations will change completely and they would not be recognisable (Fig. 1.9).

100,000 years ago **Present** **100,000 years from now**

Fig.1.9: Change in the shape of the Big Dipper due to peculiar velocities

You may now like to solve a problem based on these concepts.

SAQ 5

Spend 2 min.

The star η CMa is at a distance of 800 pc. If the proper motion of the star is 0.008″/yr, calculate its transverse velocity in km s^{-1}.

So far you have learnt how we can find distances of stars. In astronomy, it is equally important to know the sizes of stars. Are they all the same size, or are some of them smaller or larger than the others? Let us now find out how stellar radii may be measured.

1.4.2 Stellar Radii

There are several ways of measuring the radii of stars. Here we describe two methods:

- the **direct** method, and
- the **indirect** method.

Direct Method

We use this method to measure the radius of an object that is in the form of a disc. In this method, we measure the angular diameter and the distance of the object from the place of observation (see Fig. 1.10).

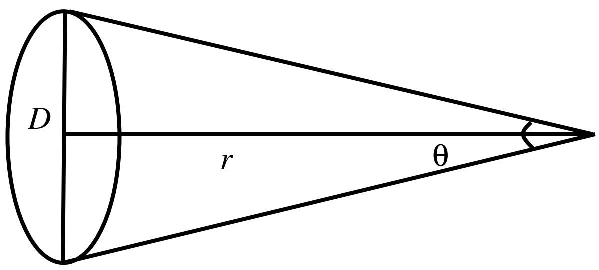


Fig.1.10: Direct method

If θ (rad) is the angular diameter and r is the distance of the object from the observer then the diameter of the stellar object will be

$$D = \theta \times r \qquad (1.12)$$

This method is useful for determining the radii of the Sun, the planets and their satellites. Since stars are so far that they cannot be seen as discs even with the largest telescopes, this method cannot be used to find their radii. For this we use other methods.

In Table 1.5 we give the radius of some stars.

Table 1.5: Radius of some stars

Star	θ (in arc seconds)	Radius (in R_Θ)
α Tau	0.020	48
α Ori	0.034	214
α Sco	0.028	187

The luminosity of a star can also reveal its size since it depends on the surface area and temperature of star. This provides a basis for the indirect method of determining stellar radii.

Indirect Method

To obtain stellar radii, we can also use Stefan-Boltzmann law of radiation

$$F = \sigma T^4 \tag{1.13}$$

where F is the radiant flux from the surface of the object, σ, Stefan's constant and T, the surface temperature of the star. You have learnt in Sec. 1.3 that the luminosity L of a star is defined as the total energy radiated by the star per second. Since $4\pi R^2$ is the surface area, we can write

$$L = 4\pi R^2 F$$

where R is the radius of the star. If the star's surface temperature is T, using Eq. (1.13), we obtain

$$L = 4\pi R^2 \sigma T^4 \tag{1.14}$$

The knowledge of L and T gives R.

Now let us consider two stars of radii R_1 and R_2 and surface temperatures T_1 and T_2, respectively. The ratio of luminosities of these two stars will be

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 \tag{1.15}$$

But from Eq. (1.8), we have

$$\frac{L_2}{L_1} = 100^{(M_1 - M_2)/5} \tag{1.16}$$

where M_1 and M_2 are the absolute magnitudes. Therefore, from Eqs. (1.15) and (1.16), we get

$$\frac{R_2^2 T_2^4}{R_1^2 T_1^4} = 10^{0.4(M_1 - M_2)} \tag{1.17}$$

Using Eq. (1.17) let us now determine the ratio of radii of Sirius A and Sirius B.

Example 2: Determining stellar radii

The surface temperatures of Sirius A and Sirius B are found to be equal. The absolute magnitude of Sirius B is larger than that of Sirius A by 10. Thus, $M_1 - M_2 = -10$ and we have

$$R_2 = 0.01 \times R_1$$

Thus the radius of Sirius A is 100 times that of Sirius B.

You may like to attempt an exercise now.

Spend 5 min.

SAQ 6

The luminosity of a star is 40 times that of the Sun and its temperature is twice as much. Determine the radius of the star.

Mass is also a fundamental property of a star, like its luminosity and its radius. Unfortunately, mass of a single star cannot be found directly. If, however, two stars revolve round each other, it is possible to estimate their masses by the application of Kepler's laws.

1.4.3 Masses of Stars

Two stars revolving around each other form a *binary system*. Fortunately, a large fraction of stars are in binary systems and therefore their masses can be determined. Binary stars can be of three kinds:

1. **Visual binary stars:** These stars can be seen moving around each other with the help of a telescope. If both the stars have comparable masses, then both revolve around their common centre of mass in elliptic orbits. If, however, one is much more massive than the other, then the less massive star executes an elliptic orbit around the more massive star (Fig.1.11).

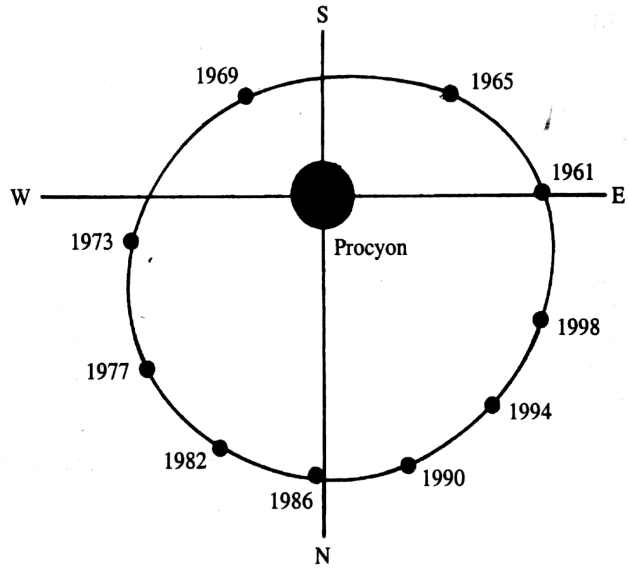


Fig.1.11: Orbit of visual binary stars

2. **Spectroscopic binary stars:** The nature of these stars being binary is revealed by the oscillating lines in their spectra. Consider the situation in Fig.1.12a. Here star 1 is moving towards the observer and star 2 is moving away from the observer. The spectral lines of star 1 are, therefore, shifted towards blue region from their original position due to Doppler Effect. The lines of star 2 are shifted towards red. Half a period later, star 1 is moving away from the observer and star 2 is moving towards the observer (Fig.1.12b). Now the spectral lines of the two stars are shifted in the directions opposite to the earlier case. In this way the spectral lines oscillate.

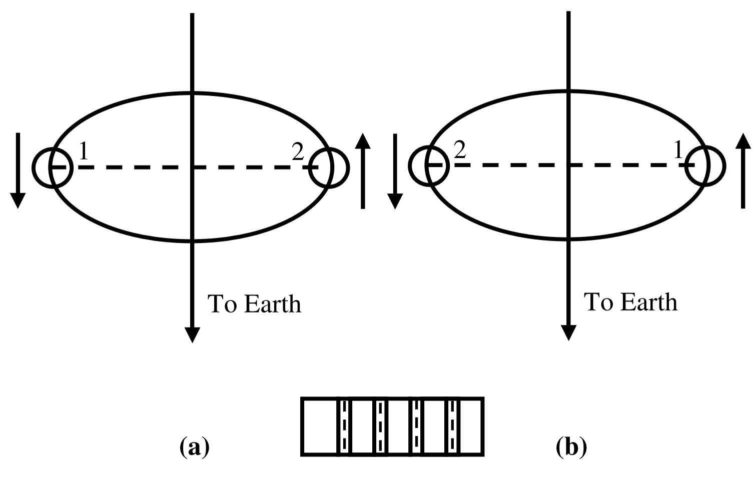


Fig.1.12: Spectroscopic binary stars

Observations of oscillating lines indicate that the stars are binary stars. If only one of the stars is bright, then only one set of oscillating lines is observed. If both the stars are bright, then two sets of oscillating lines are seen.

3. **Eclipsing binary stars:** If the orbits of two stars are such that the stars pass in front of each other as seen by an observer (Fig.1.13), then the light from the group dips periodically. The periodic dips reveal not only the binary nature of the stars, but also give information about their luminosities and sizes.

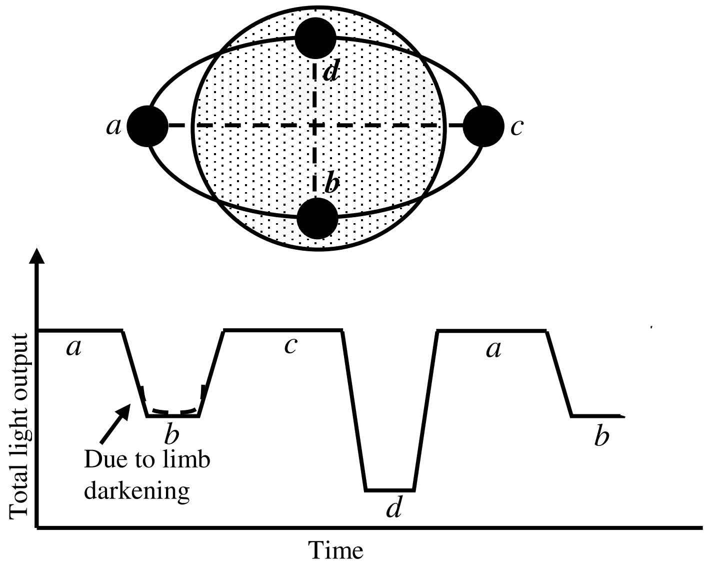


Fig.1.13: Eclipsing binary stars

Now suppose M_1 and M_2 are the masses of the two stars and a is the distance between them, then we can write Kepler's third law as

$$\frac{GP^2}{4\pi^2}\left(M_1 + M_2\right) = a^3 \qquad (1.18)$$

where P is the period of the binary system and G is the constant of gravitation. This relation gives us the combined mass of the two stars. However, if the motion of both the stars around the common centre of mass can be observed, then we have

$$M_1 a_1 = M_2 a_2 \qquad (1.19)$$

where a_1 and a_2 are distances from the centre of mass. Then both these equations allow us to estimate the masses of both the stars.

Masses of stars are expressed in units of the solar mass, $M_\Theta = 2 \times 10^{30}$ kg. Most stars have masses between 0.1 M_Θ and 10 M_Θ. A small fraction of stars may have masses of 50 M_Θ or 100 M_Θ.

So far we have discussed the ways of measuring stellar parameters such as distance, luminosity, radii and mass. Stellar temperature is another important property of a star.

1.4.4 Stellar Temperature

The temperature of a star can be determined by looking at its spectrum or colour. The radiant flux (F_λ) at various wavelengths (λ) is shown in Fig.1.14. This figure is quite similar to the one obtained for a black body at a certain temperature. Assuming the star to be radiating as a black body, it is possible to fit in a Planck's curve to the observed data at temperature T. This temperature determines the colour of the star.

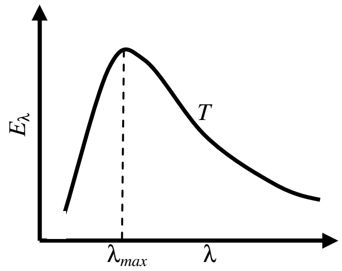


Fig.1.14: Total energy flux at different wavelengths.

The temperature of a star (corresponding to a black body) may be estimated using Wien's law:

$$\lambda_{max} T = 0.29 \text{ cm K} \tag{1.20}$$

Such a temperature is termed as surface temperature, T_s. In general it is difficult to define the temperature of a star. For instance the temperature obtained from line emission is indicative of temperature from a region of a star where these lines are formed. Similarly the effective temperature of a star corresponds to the one obtained using Stefan-Boltzman law, i.e., $F = \sigma T_e^4$.

In Table 1.6 we give the range of values of stellar parameters of interest in astronomy such as mass, radius, luminosity and stellar temperature.

Table 1.6: Range of Stellar Parameters

Stellar Parameters	Range
Mass	$0.1 - 100\ M_\Theta$
Radius	$0.01^* - 1000\ R_\Theta$
Luminosity	$10^{-5} - 10^{5}\ L_\Theta$
Surface Temperature	3000 – 50,000 K

*It is difficult to put any lower limit on the radii of stars. As you will learn later, a neutron star has a radius of only 10 km. The radius of a black hole cannot be defined in the usual sense.

We can find various empirical relationships among different stellar parameters, e.g., mass, radius, luminosity, effective temperatures, etc. Observations show that the luminosity of stars depends on their mass. We find that the larger the mass of a star, the more luminous it is. For most stars, the mass and luminosity are related as

$$\frac{L}{L_\Theta} = \left(\frac{M}{M_\Theta}\right)^{3.5} \tag{1.21}$$

In this unit we have introduced you to a number of astronomical quantities and described some simple ways of measuring them. We now summarise the contents of this unit.

1.5 SUMMARY

- The astronomical **units** of **distance**, **size**, **mass** and **luminosity** are defined as follows:
  - 1 **astronomical unit** (AU) is the mean distance between the Sun and the Earth. 1 AU = 1.496×10^{11}m.
  - 1 **light year** (ly) is the distance travelled by light in one year. 1 ly = 9.4605×10^{15}m = 6.32×10^{4} AU.

- 1 **parsec** (pc) is defined as the distance at which the radius of Earth's orbit subtends an angle of 1″. 1 pc = 3.262 ly = 2.06×10^5AU = 3.084×10^{16}m.
- 1 **solar radius** $R_\Theta = 7 \times 10^8$ m.
- 1 **solar mass** $M_\Theta = 2 \times 10^{30}$ kg.
- 1 **solar luminosity** $L_\Theta = 4 \times 10^{26}$ W.

- **Apparent magnitude** of an astronomical object is a measure of how bright it **appears**. Its **absolute magnitude** is defined as its apparent magnitude if it were at a distance of 10 pc from us.
- The difference in apparent and absolute magnitude is called the **distance modulus** and is a measure of the distance of an astronomical object:

$$m - M = 5\log_{10}\left(\frac{r\text{pc}}{10\text{pc}}\right)$$

- **Radiant flux** is the total amount of energy flowing per unit time per unit area oriented normal to the direction of its propagation. The **luminosity** of a body is defined as the total energy radiated per unit time by it.
- **Brightness** and **radiant flux** of an object are related to its apparent magnitude as follows:

$$\frac{b_2}{b_1} = 100^{(m_1 - m_2)/5}$$

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}$$

- The **absolute magnitude** and **luminosity** are related as follows:

$$M_1 - M_2 = 2.5\log_{10}\left(\frac{L_2}{L_1}\right)$$

- If θ is the parallax of an object in arc seconds, then its **distance** in parsecs is given by

$$r = \frac{1\,\text{AU}}{\theta}$$

- The motion of an object can be resolved into two components: **radial motion** and **proper motion**. The proper motion μ of a star is related to its **transverse velocity** v_θ as follows:

$$\mu = \frac{v_\theta}{r}$$

where r is its distance.

- **Stellar radii** are related to the absolute magnitudes and temperatures of stars:

$$\frac{R_2^2 T_2^4}{R_1^2 T_1^4} = \text{antilog}\{+\,0.4(M_1 - M_2)\}$$

- The masses M_1 and M_2 of stars in a binary system can be estimated from the following relations:

$$\frac{GP^2}{4\pi^2}(M_1 + M_2) = a^3$$

$$M_1 a_1 = M_2 a_2$$

where P is the time period of the binary system, G the constant of gravitation, a the distance between them and a_1, a_2, their distances from the centre of mass, respectively.

- The temperature of a star can be estimated by fitting observed data to Planck's black body radiation curve or using Wein's law: $\lambda_{max} T = 0.29\,\text{cm}$

  The temperature T of a star can also be estimated from Stefan-Boltzmann law:

$$F = \sigma T^4.$$

1.6 TERMINAL QUESTIONS *Spend* 30 *min.*

1. The apparent magnitude of full moon is – 12.5 and that of Venus at its brightest is – 4.0. Which is brighter and by how much?
2. The apparent magnitude of the Sun is – 26.8. Find its absolute magnitude. Remember that the distance between the Sun & the Earth is 1.5×10^{13} cm.
3. After about 5 billion years the Sun is expected to swell to 200 times its present size. If its temperature becomes half of what it is today, find the change in its absolute magnitude.
4. The mass of star Sirius is thrice that of the Sun. Find the ratio of their luminosities and the difference in their absolute magnitudes. Taking the absolute magnitude of the Sun as 5, find the absolute magnitude of Sirius.

1.7 SOLUTIONS AND ANSWERS

Self Assessment Questions (SAQs)

1. a) Jupiter is 5 AU from the Sun.

   1 pc = 2.06×10^5 AU

   5 AU = 2.43×10^{-5} pc

   Distance between Earth and the Sun = 1 AU.

   1 ly = 6240 AU

   1 AU = 1.6×10^{-4} ly

   b) The radius of the Earth is 0.01 R_Θ.

2. a) The Sun is brighter

   $m_{Sun} - m_{Si} = -26.81 - (-1.47) = -25.34$

$$\frac{b_1}{b_2} = 100^{25.34/5} = 100^{5.07} = 1.38 \times 10^{10}$$

The Sun is about 10^{10} times brighter than Sirius.

b) $$\frac{b_1}{b_2} = 100^{-(0.06-0.86)/5} = 100^{+.80/5} = (10)^{0.32} = 2.09$$

3. a) $$m - M = -0.5 = 5 \log_{10}\left(\frac{r}{10\,\text{pc}}\right)$$

$$\log_{10}\left(\frac{r}{10\,\text{pc}}\right) = \left(-\frac{0.5}{5}\right) = -0.1$$

$$\frac{r}{10\,\text{pc}} = (10)^{-0.1}$$

$r = 7.9$ pc

b) $$\frac{F_2}{F_1} = \left(\frac{40}{10}\right)^2 = 16$$

It will appear 16 times brighter, which corresponds to $m = 3$ from Table 1.2.

4. a) $r = \frac{1\,\text{AU}}{\theta}$ pc ;

Alpha Centauri 1.34 pc;

Barnard's star 1.81 pc;

Altair 5.07 pc;

Alpha Draco 5.68 pc

b) Distance = 500 pc

5. $v_\theta = 4.74\ \mu\ r = 30.34$ km s^{-1}

6. $M_1 - M_2 = 40$

$$\frac{R_2^2}{R_1^2} = \frac{T_1^4}{T_2^4} . \frac{L_2}{L_1} = \left(\frac{1}{2}\right)^4 \times 40$$

$$R_2^2 = \left(R_1^2\right) \times 2.5$$

$$R_2 = 1.58 R_\Theta .$$

Terminal Questions

1. Apparent magnitude of moon is lower (larger negative number), than that of Venus. Therefore, moon is brighter than Venus.

   Moreover,

$$\frac{b_{\text{moon}}}{b_{\text{venus}}} = 10^{-0.4(m_{\text{moon}} - m_{\text{venus}})}$$

$$= 10^{-0.4(-12.5+4.0)} \quad = 10^{-0.4-8.5} = 10^{3.4} \quad = 2.5\times 10^{3}.$$

2. The relation between apparent magnitude m and absolute magnitude M is

$$M = m - 5 \log r + 5$$

   where the distance r is in parsec. Distance of the Sun in parsec is $1.5 \times 10^{13}/3 \times 10^{18} = 5 \times 10^{-6}$. So,

$$M = -26.8 - 5 (\log 5 - 6) + 5 = -26.8 - 5 \log 5 + 30 + 5$$

$$= 8.4 - 3.5 = 4.9$$

3. According to Eq. (1.17)

$$\frac{(200)^2}{(2)^4} = 10^{0.4(M_1 - M_2)}$$

   where M_1 is the present absolute magnitude of the Sun. Therefore,

$$M_1 - M_2 = 2.5 \log\left(\frac{200\times 200}{16}\right)$$

$$= 2.5 \log (2500) = 2.5 \times 3.4 = 8.5$$

   So, the absolute magnitude of the Sun will decrease by 8.5 and it will, therefore, become much more luminous.

4. Using Eq. (1.21)

$$\frac{L_{\text{Sirius}}}{L_{\Theta}} = (3)^{3.5}$$

Now using Eq. (1.16),

$$(3)^{3.5} = 10^{0.4(M_\Theta - M_{Sirius})}$$

where M_Θ and M_{Sirius} are absolute magnitudes of the Sun and Sirius.

So, $(M_\Theta - M_{\text{Sirius}}) = 2.5 \log (46.8)$

$= 2.5 \times 1.7 = 4.25.$

$\therefore \quad M_{\text{Sirius}} = 5 - 4.25 = 0.75.$